

AD-A166 958

THREE-DIMENSIONAL GEODETIC CONTROL BY INTERFEROMETRY  
WITH GPS (GLOBAL POS. (U) MASSACHUSETTS INST OF TECH  
CAMBRIDGE DEPT OF EARTH ATMOSPHERI. Y BOCK ET AL

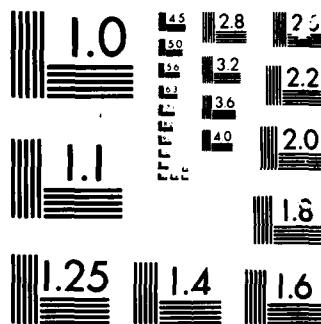
1/1

UNCLASSIFIED

23 APR 85 AFGL-TR-86-0076 F19628-82-K-0002 F/G 17/7

NL





MICROCOPY

CHART

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

4

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFGL-TR-86-0076	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Three-Dimensional Geodetic Control by Interferometry with GPS: Processing of GPS Phase Observables		5. TYPE OF REPORT & PERIOD COVERED Interim Technical 2/82-4/85
7. AUTHOR(s) Y. Bock, R.I. Abbot, C.C. Counselman III, R.W. King, S.A. Gourevitch		6. PERFORMING ORG. REPORT NUMBER none
9. PERFORMING ORGANIZATION NAME AND ADDRESS Massachusetts Institute of Technology Dept. of Earth, Atmospheric and Planetary Sciences Cambridge, MA 02139		8. CONTRACT OR GRANT NUMBER(s) F19628-82-K-0002
11. CONTROLLING OFFICE NAME AND ADDRESS AFGL/LWG Hanscom Air Force Base, MA 01731		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2309G1BD
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) same		12. REPORT DATE 4/23/85
		13. NUMBER OF PAGES 8
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) <div style="text-align: right;">DTIC ELECTE APR 28 1986 S B D</div>		
18. SUPPLEMENTARY NOTES Submitted for publication in the Proceedings of the First International Symposium on Precise Positioning with the Global Positioning System, April 15-19, 1985, Rockville, MD		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) geodesy; geodetic control; geodetic networks; three-dimensional geodesy; satellite geodesy; NAVSTAR Global Positioning System; GPS; interferometry		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) See reverse side.		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

AD-A166 958

DTIC FILE COPY

↓

ABSTRACT. Interferometry with the NAVSTAR Global Positioning System (GPS) is the most efficient method of establishing three-dimensional geodetic control on local and regional scales. This is already true, even though the constellation of satellites is incomplete. We consider some theoretical and practical aspects of using differenced carrier-phase observations of the GPS satellites to establish three-dimensional control networks. We present simple and efficient algorithms for processing multi-station, multi-satellite observations. These algorithms have been used to establish three-dimensional control networks. Under widely varying conditions, we have obtained accuracies of 1 to 1.5 parts per million (ppm) in all three coordinates, or 1 to 3 cm for the coordinate differences between network stations separated by 10 to 20 km. When interferometry is used also to determine the satellite orbits, we anticipate improvement of the accuracy of regional control networks to the level of 0.1 ppm.

Accession No.	
NTIS	✓
DTIC	
Unrec.	
Dist.	
By	
Dist.	
Avail.	
Dist./or	
Dist.	
A-1	

# AFGL-TR-86-0076

## THREE-DIMENSIONAL GEODETIC CONTROL BY INTERFEROMETRY WITH GPS: PROCESSING OF GPS PHASE OBSERVABLES

Y. Bock, R.I. Abbot, C.C. Counselman III, R.W. King  
Dept. Of Earth Atmospheric and Planetary Sciences  
Massachusetts Institute of Technology  
Cambridge, MA 02139

S.A. Gourevitch  
Steinbrecher Corporation  
185 New Boston St.  
Woburn, MA 01801

**ABSTRACT.** Interferometry with the NAVSTAR Global Positioning System (GPS) is the most efficient method of establishing three-dimensional geodetic control on local and regional scales. This is already true, even though the constellation of satellites is incomplete. We consider some theoretical and practical aspects of using differenced carrier-phase observations of the GPS satellites to establish three-dimensional control networks. We present simple and efficient algorithms for processing multi-station, multi-satellite observations. These algorithms have been used to establish three-dimensional control networks. Under widely varying conditions, we have obtained accuracies of 1 to 1.5 parts per million (ppm) in all three coordinates, or 1 to 3 cm for the coordinate differences between network stations separated by 10 to 20 km. When inteferometry is used also to determine the satellite orbits, we anticipate improvement of the accuracy of regional control networks to the level of 0.1 ppm.

First International Symposium on Precise Positioning  
with the Global Positioning System  
Rockville, Maryland  
April 15-19, 1985

## INTRODUCTION

GPS interferometry is a method by which three-dimensional relative-position vectors between observing stations can be estimated with respect to a world-wide, crust-fixed coordinate system. The processing of GPS observations and the accuracies obtained under widely varying conditions, not only for relative-positioning but also for single-point positioning and for GPS satellite orbit determination, have been described by Counselman et al. (1983), Bock et al. (1984), King et al. (1984), Bock et al. (1985), Abbot et al. (these proceedings), and Ladd et al. (these proceedings).

We assume in this paper that the basic observable is a precise measurement of the difference between the phase of the reconstructed carrier wave of the signal received from a GPS satellite, and the phase of a reference signal generated within the receiver at one site, at a series of epochs determined by the clock in the receiver. We call this observable a one-way phase. (Such an observation can be made for the L1 and/or L2 frequency band.) In practice, for a particular observing session, a number of receivers simultaneously observe several satellites over a time span ranging from several minutes to several hours. The simultaneity of observations is crucial since many sources of error tend to affect different observations equally at a given time, particularly instabilities in the receivers' and the satellites' oscillators, and errors in models of the satellite orbits and the propagation media. Differencing of the simultaneous observations at different stations cancels common-mode errors. Satellite-oscillator errors are virtually eliminated, and the residual errors of both atmospheric and orbital origin are approximately proportional to the distance between the stations, up to several tens of kilometers. We call the resulting observables single-differences. Differencing again, between satellites that were observed simultaneously, results in an observable that is also free of receiver oscillator instability. The ability to observe several satellites simultaneously, and therefore to form double-differences, is what makes GPS so powerful for relative-position determination. This fact should not be forgotten in discussions of so-called "undifferenced" processing methods.

In this paper, we consider some theoretical and practical problems in the use of multi-station, doubly-differenced phase, observations to establish three-dimensional geodetic control. We present our solutions to these problems in the form of simple and efficient algorithms. A companion paper describes the application of these ideas to GPS satellite orbit determination (Abbot, et al., these proceedings).

## DATA PROCESSING ALGORITHMS

Consider a typical GPS observing session in which M stations observe up to N satellites simultaneously. Satellites rise and set throughout the observation span. In addition, temporary obstructions of the satellite signals at one or more of the tracking sites may cause the number of one-way phase observations to differ from site to site.

Let us assume for a moment that there are no losses of data so that matching sets of observations are collected at the sites. From the one-way phases, at each epoch one can form  $M(M-1)/2$  single-differences, but only  $M-1$  are independent. The single-differences will sum to zero around any loop of stations, regardless of the nature of the observation errors; i.e., the closure is trivial. If each baseline were estimated independently, the baseline vectors would sum to zero around each closed figure.

Differencing the one-way phases between stations introduces 50% correlations between the single-differences at an epoch, in the simple case of equal-variance uncorrelated errors at the individual sites. If these correlations are accounted for, any choice of  $M-1$  single differences will yield identical baseline estimates.

With this point in mind, consider the typical observing session where the observation sets differ from site to site. The following two-step algorithm will extract all the information from the raw observables and at the same time will take into account the correlations that appear as a result of differencing the one-way phases.

The basic (linearized) observation equations for one-way phases can be expressed in familiar matrix form by

$$\phi = AX + V + V_c \quad (1)$$

where  $\phi$  is the observation vector containing the differences between the observed phases and their a priori computed values,  $A$  is the design matrix containing the partial derivatives of the phases with respect to the parameters of interest,  $X$  is the parameter correction vector,  $V$  is the vector of the non-common-mode observation errors, and  $V_c$  is the vector of the common-mode observation errors. For the estimation model, we assume for the (normalized) non-common-mode observation errors that

$$E\{V\} = 0 \quad (2)$$

$$E\{VV^T\} = I \quad (3)$$

where  $E$  denotes expectation and  $I$  is the identity matrix. The common-mode observation errors cancel in the subsequent differencing and can be neglected. The assumption, therefore, is that the non-common-mode errors of the one-way phases are statistically independent in space and time, with equal (and normalized) errors. The parameters of interest for three-dimensional control applications are discussed in the next section.

Let us denote the one-way phase for station  $m$  and satellite  $n$  by  $\phi(m,n)$ . At each epoch,  $M-1$  uncorrelated and normalized single-differences can be computed for each of the  $N$  satellites by

$$DI(m,n) = \left[ \phi(m+1,n) - \frac{1}{m} \sum_{m=1}^{M-1} \phi(m,n) \right] \left( \frac{m}{m+1} \right)^{1/2} \quad (4)$$

using a Gram-Schmidt orthogonalization scheme. (Note that the index  $m$  in  $D1(m,n)$  denotes only the numbering of the single-differences, whereas on the right-hand side of (4),  $m$  denotes the station number.)

For example, for 4 stations observing simultaneously the signals from satellite 1,

$$D1(1,1) = [\phi(2,1) - \phi(1,1)] \left(\frac{1}{2}\right)^{1/2} \quad (5)$$

$$D1(2,1) = [\phi(3,1) - \frac{1}{2}(\phi(1,1) + \phi(2,1))] \left(\frac{2}{3}\right)^{1/2} \quad (6)$$

$$D1(3,1) = [\phi(4,1) - \frac{1}{3}(\phi(1,1) + \phi(2,1) + \phi(3,1))] \left(\frac{3}{4}\right)^{1/2}. \quad (7)$$

It can be easily verified that under the assumptions (2-3), the observation errors of this set of single-differences are uncorrelated (and normalized).

Suppose, that at some epoch the view of station 1 to satellite 1 was obstructed so that  $\phi(1,1)$  was unavailable. Then only  $M-2=2$  uncorrelated single-differences could be formed for satellite 1 from the one-way phases collected at the three remaining sites by

$$D1(1,1) = [\phi(3,1) - \phi(2,1)] \left(\frac{1}{2}\right)^{1/2} \quad (8)$$

$$D1(2,1) = [\phi(4,1) - \frac{1}{2}(\phi(2,1) + \phi(3,1))] \left(\frac{2}{3}\right)^{1/2}. \quad (9)$$

This simple scheme ensures that all one-way phase observations are used, except in the case when only one site is observing at an epoch.

However, the one-way phases contain information on the absolute positions of the observing sites, as well. Therefore, if we include at each epoch the common-mode observable for each satellite

$$\bar{\phi}(n) = \frac{1}{M} \sum_{m=1}^M \phi(m,n) \quad (10)$$



then the coordinates of each observing station can be estimated with respect to a global terrestrial reference frame. Of course, the relative positions of the sites will be more accurately estimated than the absolute positions (see Bock et al., 1984). Note that the common-mode observable is orthogonal to the single-differences. For receivers with crystal oscillators, a between-satellites observable (see below) is more valuable for point-positioning since any instabilities in the receivers' oscillators are eliminated. The one-way phase common-mode observable is valuable for more accurate frequency standards (e.g., cesium-beam).

For the most accurate baseline determination, a second difference is taken, between satellites. The above orthonormalization scheme can be applied again. The double-differences are denoted by  $D2(m,n)$  where  $n$  is an index that runs from 1,  $N-1$ , where  $N$  is the number of satellites observed simultaneously by the  $M$  stations. We form for each set  $m$  (see (4)) of single-differences

$$D2(m,n) = \left[ D1(m,n+1) - \frac{1}{n} \sum_{n=1}^{N-1} D1(m,n) \right] \left( \frac{n}{n+1} \right)^{1/2}. \quad (11)$$

(Note that the index  $n$  in  $D2(m,n)$  denotes only the numbering of the double-differences, whereas on the right-hand side of (11),  $n$  denotes the satellite number.) Thus, for  $M$  stations observing  $N$  satellites, there are  $(M-1)(N-1)$  orthonormalized double-differences that can be formed per epoch. This scheme is applied to all visible satellites, so that risings and settings of satellites are handled naturally (as obstructions are handled in the first orthonormalization described above). Note that the order of orthonormalization could be reversed. That is, the satellite orthonormalization could precede the station orthonormalization. The latter order may be preferred for point-positioning with satellite-differenced observations.

#### PARAMETER ESTIMATION

The mathematical model relating the one-way phases to the geodetic parameters can be found elsewhere (e.g., Fell, 1980; Remondi, 1984). Here we point out that the partial derivatives used in a weighted least-squares fit of the orthonormalized double-differences are manipulated in the same way as the observables in (4) and (11) (i.e., they are also orthonormalized as the normal equations are being filled).

In three-dimensional network applications, for  $M$  stations observing a total of  $N$  satellites, we adjust as a minimum  $3(M-1)$  station coordinates ( $3M$  if we are also point-positioning using one-way or satellite-differenced phases) and  $(M-1)(N-1)$  phase-bias parameters. The phase-bias parameters represent the ambiguities of the one-way phase observables. If the receiver clocks were not synchronized and we do not know the departure from synchronization a priori (e.g., from field measurements), then we solve for  $M-1$  clock offset parameters as well. Rate and

higher order clock terms cannot be determined well from the double-differences. They are better determined from the single-differences.

Theoretically, each of the double-difference phase-bias parameters has an integer value. (The value would be zero but for the intrinsic ambiguity of the phase observable -- that is, the indistinguishability of the cycles of a periodic signal). In general, the most accurate baseline estimates are obtained if these biases can be constrained to the correct integer values. If the statistical significance, or level of confidence, of this integer determination is sufficient, we adjust again with the bias parameters fixed at the best-fitting integer values. We call this a biases-fixed solution. Otherwise, we consider the preliminary adjustment with its real-valued biases as final. We call this a biases-free solution.

With a typical, three- to five-hour-long, schedule of observations from single-band GPS instruments, and with the present 0.5- to 1-ppm level of uncertainty in the satellite orbits, we find that we can fix biases when the baseline length is up to 20 or 30 km. Strange (1984) has reported success in fixing biases on baselines of up to 50 km in length. With improved satellite ephemerides and/or with the use of dual-band receiving instruments, fixing biases is possible for longer baselines and for shorter observing schedules.

#### PRACTICAL DIFFICULTIES AND SOLUTIONS

A practical difficulty with double-orthonormalization in the multi-station and multi-satellite mode stems from the occurrence of cycle-slips in the one-way phase measurements. A cycle-slip is an occasional, sudden, gain or loss of some whole number of cycles, due, for example, to temporary occultation of a satellite. If the slip is not corrected in the data processing by the addition of the correct integer number of cycles for the appropriate station, satellite, epoch (and frequency band in the case of dual-band observations), then obviously the geodetic position estimates will be corrupted.

The ability to recognize a cycle-slip and to be certain of "fixing" it depends on the magnitude of the "noise" in the observations, relative to the magnitude of one cycle. For a baseline of the order of 10 km, for example, the r.m.s noise level in the double-difference observations is typically about 0.1 cycle. Therefore a slip of just one cycle is conspicuous in a series of double-difference observations from a single-pair of satellites and a single-pair of stations. But if observations of multiple satellites from multiple stations are linearly combined in a double-orthonormalization, a single-cycle slip in one of the original observations becomes mapped into fractional-cycle-slips in many new, orthonormal observations. With many satellites and many stations, the number of combinations is enormous.

Our approach to the problem of cycle-slips is to introduce automatically, in a preliminary adjustment, a new phase-bias parameter whenever a gap is encountered in the phase data. Most cycle-slips (particularly the larger ones) are associated with losses of data (gaps) at one or more epochs. The post-fit residuals of the (simple, not orthonormalized) double-differences indicate the exact epoch,

satellite and frequency band where a cycle-slip occurred. (This preliminary adjustment also serves to provide good starting estimates for the baseline vector components.) Now the cycle-slips are fixed appropriately in the one-way phase data and a second adjustment is performed using the double-orthonormalization algorithm on the cycle-slip-free data. The integer phase-biases are then fixed, if possible, as discussed earlier.

Note that it is necessary to double-difference the phase data (1) in order to correct the one-way phases for cycle-slips (since the noise level is sufficiently low to detect the cycle-slips only in double-differences) and (2) in order to fix the phase-biases at their best-fitting values (since the biases are integers only after double-differencing).

For an application of the approach outlined in this paper, see the paper by Bock et al. (1985), in which the analysis of a 35-station three-dimensional geodetic network is described. The results of that analysis indicate that three-dimensional geodetic control can be routinely obtained at the 1-2 ppm level under widely varying conditions. When interferometry is used also to determine the satellite orbits, we anticipate improvement of the accuracy of regional geodetic control to the level of 0.1 ppm.

#### ACKNOWLEDGEMENT

Research at M.I.T. has been supported by the U.S. Air Force Geophysics Laboratory (AFGL), Geodesy and Gravity Branch, under contract F19628-82-K-0002.

## REFERENCES

- Abbot, R.I., Y. Bock, C.C. Counselman, R.W. King, S.A. Gourevitch and B. Rosen, 1985: Interferometric Determinations of GPS Satellite Orbits, these proceedings.
- Bock, Y., R.I. Abbot, C.C. Counselman, S.A. Gourevitch, R.W. King and A.R. Paradis, 1984: Geodetic Accuracy of the Macrometer<sup>tm</sup> Model V-1000, Bulletin Geodesique 58, 211-221.
- Bock, Y., R.I. Abbot, C.C. Counselman, S.A. Gourevitch and R.W. King, 1985: Establishment of Three-Dimensional Geodetic Control by Interferometry with the Global Positioning System, Journal of Geophysical Research (Solid Earth), in press.
- Counselman, C.C., R.I. Abbot, S.A. Gourevitch, R.W. King and A.R. Paradis, 1983: Centimeter-level Relative Positioning with GPS, Journal of Surveying Engineering, 109, 81-89.
- Fell, P.J., 1980: Geodetic Positioning Using a Global System of Satellites, Dept. of Geodetic Science and Surveying Rep. 299, The Ohio State University, Columbus, Ohio.
- Gourevitch, S.A., 1984: The GPS Interferometric Observable, M.I.T. internal memorandum.
- King, R.W., R.I. Abbot, C.C. Counsleman, S.A. Gourevitch, B.J. Rosen and Y. Bock, 1984: Interferometric Determination of GPS Satellite Orbits, EOS, Trans. AGU, 65, 853.
- Ladd, J., C.C. Counselman and S.A. Gourevitch, 1985: The Macrometer II<sup>tm</sup> Dual-Band Interferometric Surveyor, these proceedings.
- Remondi, B.W., 1984: Using the Global Positioning System (GPS) Phase Observable for Relative Geodesy: Modeling, Processing, and Results, Report CSR-84-2, Center for Space Research, Univ. of Texas, Austin.
- Strange, W.E., 1984: The Accuracy of Global Positioning System for Strain Monitoring, EOS, Trans. AGU, 65, 854.

END  
FILMED

5-86

DTIC